### COMPUTATIONAL APPROACHES TO APPLIED MATHEMATICAL MODELING BRIDGING THEORY AND REAL-WORLD APPLICATIONS

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#### Abstract

Applied mathematical modeling is solely dependent on computational techniques for the integration of math and the real world through vibrant analysis, optimization and prediction for intricate systems. The gap between theory and practical application is bridged through advanced numerical simulation methods, like Monte Carlo simulations, finite element analysis, or machine learning based solvers, which have remarkably improved the efficiency and accuracy of mathematical models. The distributed computing architecture and GPU based solvers enable high performance computing, which has made large scale simulations for engineering, physics, epidemiology, finance, environmental sciences and even medicine possible. On the other hand, computational modeling leads to new challenges like the curse of dimensionality, algorithmic efficiency bottlenecks, and instability-accuracy compromises in numerical methods. In contrast, solving mathematical problems bring unbounded promises though quantum computing, topological data analysis, and AI powered solvers. They will build almost indisputable freedom when it comes to accuracy and scalability within the future frameworks of mathematics. Constant adaptation frameworks and real-time data assimilation is revolutionizing predictive analytics within medicine and climate modeling. Solving problems of practical significance becomes easier through cross domain synergies where decisions optimized by resource allocation and powered by computational mathematics improves the impact along with transforming scientific research. Applied mathematical modeling will continue to be useful in the scope of scientific and technological development in fostering new discoveries and practical advances due to new computational capabilities after innovations on a variety of fields.

### INTRODUCTION

The applied mathematical modeling is essential in the transformation of human activities into real world phenomena as it helps in creating structures for analysis and forecasting (Ballard et al., 2021). This concept also takes into account the building of automated systems as an aspect of <u>construction</u>, biology, and economy by applying differential equations, linear algebra, and optimization methods. A case in point is population dynamics, which can be simplified by the logistic growth formula. dt/dP = rP(1-K/P)



Where P is the population size, r is the intrinsic rate of natural increase (e.g., for some bacterial populations, r = 0.02), and K is the prize of exploitable resources (e.g., K=500 in environments with limited resources) (Smith et al., 2024). Simple models like these have easier analytical solutions. However, the real world is never that easy due to the fact that numerous systems within systems result in systems experiencing nonlinear interactions with higher dimensions. This makes it almost impossible to achieve closed form solutions and the Navierstokes equations which control fluid dynamics is a good example of this complexity:

### $\rho(\partial t \partial u + u \cdot \nabla u) = -\nabla p + \mu \nabla 2u + f$

Where  $\rho$  is the fluid density, like 1000 kg/m<sup>3</sup> for water, u is the fluid velocity, p is the pressure, and  $\mu$ \mu\mu is the dynamic viscosity, 0.001 Pa for water (Haasler et al., 2021). Because of their nonlinearity, analytical solution is not possible for these equations, so they must be computed with techniques such as finite element and spectral methods.

The computation techniques enable us to solve mathematical models that are too complex to be solved analytically (Peng et al., 2021). FDM and FEM are the most popular techniques to discretize and compute differential equations. For instance, take the classic problems of heat conduction described with heat equation:

∂t∂u =α∇2u

With u(x,t) as temperature distribution and \alpha as thermal diffusivity (1.2 \* 10^{-5} m^2)/s for copper), numerical techniques like explicit forward difference method compute derivatives as

uin+1 =uin  $+\Delta x 2\alpha \Delta t$  (ui+1n -2uin +ui-1n ) This gives an easier way to calculate temperature for every instant in time (El-Emam et al., 2021). Likewise, gradient descent is one of the many numerical methods used to solve minimization problems of complex functions, like

 $xn+1 = xn -\eta \nabla f(xn)$ 

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to specify adjusts required correct me if im wrong here, how about taking 0.01 as an example for machine learning where it used as a parameter in adjusting the learning functions (Following, 2016) There exist other approaches similar that allows and requires the relevant constant functions for other disciplines like such as operating math, machine analysis and also engine functioning and constructing. Overstatement Computation error and accuracy Inaccuracy Orthography system Program Software Mathematics and optimizations They start with arrangement of concepts There is no doubt that it This is Absolutely notice observations I A B I Very Much All Are undoubtedly observed reiterated that that Boundaries mainly are attributed attributed to being associated to in relation to and about On the Achievement of Objectives for Optimizations of Set Arithmetic Program Software Though numerical models provide accurate results Multinomial Multi degree Expansion Method Multi-variable loop models Multilevel Multiscale Computational Models **Optimization Program Software Algorithm Proximity** Systems with Multi-factor Control Threshold Arithmetic Program Software Effective For models that are evaluated with various output controls, there exists a limit value These restrictions reveal focus and the boundary Between accuracy and effectiveness  $\Delta t \leq 2\alpha \Delta x^2$ 

In climate models, Petascale computing capabilities roughly estimated at 10^15 floating -point calculations per second are needed for the simulation of temperature changes using partial differential equations with 1-degree latitude and longitude grid resolution (Haasler et al., 2021). Developing quantum computing and artificial intelligence brings new possibilities, as novel methodologies can notoriously resolve problems once believed to be impossible. The managed progress of these methods will narrow the divide even more between practical use of engineered systems and their theoretical mathematical representations, making sure mathematics continues to serve as an aid for innovations in science and technology.

### Mathematical Foundations of Computational Modeling

### Differential Mathematics and Computation

The implementation of mathematics in engineering starts with numerical methods and computational modeling (Waters et al., 2021). In the context of computational mathematics, the concept of ordinary differential equations (ODE) encompasses singlevariable dependencies as in Newton's second law of motion given by:

### mdt2d2x = F(x,t)

Where  $\{m\}$  is the mass (e.g. 2kg) and xxx is displacement while F(x,t) is the external force applied. More complex systems rely on partial differential equations (PDE), for example, in thermodynamics - heat equation:

### ∂t∂u =α∇2u

Where {u}(x,t) represents the temperature while  $\alpha \geq 0$  for thermal diffusivity (for metals is  $1.2 \times 10-5 \text{ m}^2/\text{s} \geq 0$  m/2)/  $\leq (\text{m}^2)/\text{s}$  (Hansen et al., 2021). PDE's have analytical solutions that require the use of some numerical methods like finite difference (FDM) and finite element methods

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(FEM). Stability conditions like the Courant-Friedrichs-Lewy (CFL) condition places bounds on the selection of the time-step, such as:

### ∆t≤2α∆x2

Providing guarantee for ensuring accuracy in explicit schemes (Strikwerda, 2004). Furthermore, the rate of convergence of finite difference methods determines the reliability of the numerical approximations solutions, second-order schemes yield an error bound that is as follows:  $O(\Delta x^2)$ , which for the reverse problem demands high precision.

### Theory of Linear Algebra and Matrix Computation

The core numerical processes related to computer modeling rest on matrix relations. (Sun et al., 2023) Eigenvalue problems defined in terms of –  $Av=\lambda v$ 

Where A is m n matrix, the symbol  $\lambda$  denotes eigenvalues and {v} is an eigenvector, are vital in (Adcock & Dexter, 2021) structural mechanics and fluid dynamics (Hormuth et al., 2021). In a bridge functioning as an elementary finite system structural construction, the eigenvalues of the stiffness matrix determine the natural frequencies of the mechanical structures.

Sparse matrix techniques are notable for increasing computational efficiency during the execution of large-scale simulations. Direct solvers become unfeasible for large N owing to the O(N^3) Gaussian elimination's complexity, which makes iterative techniques like conjugate gradient (CG) for sparse linear systems necessary. Solving the Navier-Stokes equations in computational fluid dynamics (CFD) with spectral methods or finite volume methods require preconditioning of the matrices for better convergence rates (Llorente et al., 2023).

### Optimization Theory

Optimization techniques are well-known features in the development processes of computational models, particularly for the economic and engineering design systems (Gradient techniques such as Newton's method allow a series of improvements to be made to a solution defined from an explicit linear model by solving

 $xn+1 = xn -\eta \nabla f(xn)$ 

The parameter  $\eta$  eta is often referred to as the learning rate, and as a convention, values such as

0.01 for machine learning applications are used (Chunarkar-Patil et al., 2024). In non-linear constrained optimization problems that occur in areas like logistics or finance, Lagrange multipliers are used:

### $L(\mathbf{x},\lambda){=}f(\mathbf{x}){+}\lambda g(\mathbf{x})$

Where g(x) are constraints on the decision variables. Techniques of convex optimization, like quadratic programming, ensure global optima are found; however, non-convex problems, like training with deep learning, rely on heuristics or meta-heurics - for instance, genetic algorithms. Topology optimization in engineering is an example, which improves structural design by reducing material without compromising mechanical performance (Liu et al., 2022).

### Applications in Engineering and Economics

The use of computational strategies in applied mathematical modeling aids in addressing problems such as structural engineering, fluid dynamics, and even forecasting economics (Xiong et al., 2023). For example, in aerodynamics, aircraft designs are enhanced by running simulations based on the Euler equations, which results in lower drag coefficients. Option pricing in financial models, like the Black Scholes model, uses finite difference methods for numerical calculations as follows:

 $\partial t \partial V + 21 \sigma 2S2 \partial S2 \partial 2V + rS \partial S \partial V - rV = 0$ 

Where S is the stock price, V is the value of the option, (e.g 0.2) denotes volatility is and r (e.g. 5%) is the risk-free rate (Ghaffari Laleh et al., 2022). These technologies illustrate the usefulness of mathematical models because of their ability to integrate real world issues with modern computing, thus creating accuracy and effectiveness.

# Computational Techniques in Applied Mathematics

### Numerical Methods of Simulation

Numerical simulation techniques are important for portraying complicated real-life systems for which analytical solutions do not exist. Monte Carlo methods are among the most popular methods when stochastic modelling is concerned as they depend on random sampling to estimate probabilistic phenomena in a variety of domains such as finance, physics, and risk evaluation (Sharafati et al., 2021).

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The valuation of options contracts in finance is one example. Monte Carlo simulations estimate the expected value of payoff attached to financial derivatives by hypothesizing thousands of different possible future paths for a company's stock price and then employing a stochastic model of geometric Brownian motion.

### St+ $\Delta t$ =St e(r-21 $\sigma$ 2) $\Delta t$ + $\sigma \Delta t$ Z

Where  $S_t$  is the stock price at time t, r is the risk-free interest rate,  $\sigma$  is volatility, and Z is a standard normal variable (Biswas et al., 2021). Similarly, Lattice Boltzmann methods (LBM) have also been developed to solve fluid dynamics problems by nanoscopic discretization of the Boltzmann transport equation and make numerical simulations of airflow over aircraft wings or blood flow in arteries (Ramstead et al., 2022). In engineering, the finite element method (FEM) and boundary element method (BEM) are used to model and solve the structural and thermal analysis of spatial domains which are termed as finite elements leading to accurate solutions with considerable savings in computational resources (Sharma et al., 2022).

### Artificial Intelligence and Neural Networks

With the advent of machine learning and computational intelligence, the area of mathematical modeling has been completely transformed by allowing solutions for differential equations and optimization problems to be solved using provided data.

Neural Networks (NNs) employed in the Recurrent Neural Networks architecture promote the solution of Partial Differential Equations (PDE) problems, such as the Navier Stokes equations in fluid dynamics, by estimating complicated functions through labeled datasets (Okorie et al., 2021). As described by the universal approximation theory, any continuous function is approximated by a hidden layer AI (Artificial Intelligence) NN (Neural Network) with non-linear activation function: begin{equation}  $F(x) = \sum_{i=1}^{n} a_i = i^{n} + i^{n}$ 

### $f(x) \approx i=1 \sum N$ wi $\sigma(xi)$

Where wi are the weights, and  $\sigma$  (x) is an activation function like ReLU or sigmoid (Khaleghi et al., 2022). The optimization of decision making in non-deterministic environments where there is a

sequence of actions that may or may not take place, such as traffic control, is achieved using Reinforcement Learning (RL), where the system maximizes the total reward using the Bellman equation:  $\begin{equation} V(s) = max_a {Q(s,a)} \\ end{equation}$ 

### $Q(s,a)=r+\gamma a'max \quad Q(s',a')$

Where Q(s,a) corresponds to the action a taken in the states, r is the gained reward and \gamma is a discount factor (Ye et al., 2023). Moreover, methods of CAS type symbolic computation provide precise algebra manipulating abilities in engineering and physics by enabling the solver to construct complex equations analytically rather than numerically (Wang, 2001).

# Mathematics Parallel and High Performance Computer

The increasing components of a problem have made High Performance Computing (HPC) a necessity to efficiently solve large-scale computational mathematical models. As GPU parallel processing greatly improves the speed of matrix operations pertaining to climate simulation and biomedical imaging, it is widely used in imaging and medical procedures (Liu et al., 2022). For example, solving a linear system with a GPU-implemented Conjugate Gradient Method (CGM) solves the problem in O(n) using thousands of cores, compared to O(n^3) with traditional methods. Frameworks like MPI (Message Passing Interface) certainly allow performing realtime simulations in fluid dynamics, astrophysics, and many other fields that require solving PDEs with billions of variables (Easttom, 2022). These programs require enormous computational power which is made possible thanks to MPI.

### Applications in Weather Forecasting and Financial Modeling

Real-time computation in game modeling and advanced hydraulic modeling are especially important in game theory, weather modeling, and other fields that rely on new computations. Weather systems like forecasting the global system (GFS) will model the atmosphere and resolve its primitive equations while motion is defined as follows:  $Dt/DV = -\rho/1 \quad \nabla p + g + F$   $\{V\} \text{ is velocity, } \rho \text{ is air density, } p \text{ all represent pressure,} \\ \text{and g expresses gravity (Hormuth et al., 2021). They} \\ \text{conduct the simulation with the use of finite volume.} \\ \text{Just like that, Monte Carlo methods are} \\ \text{fields.} \\ \text{implemented in financial models when simulating} \\ \text{Applications of Computational Mathematical Modeling References} \\ \end{cases}$ 

amounts of money during fluctuations of uncertainty (Ballard et al., 2021). All of these methodologies increase precision in predictive analysis in various fields.



By providing answers to intricate real-life issues, computational mathematical modeling serves a unique purpose, as it is integral to physics, engineering and even the biomedical sciences. One of the most paramount examples from engineering and physics would be aerodynamics, weather forecasting and ocean modeling. These are advanced fields that can now be easily tackled using CFD along with the Navier-Stokes equations  $\rho(\partial t\partial v + (v \cdot \nabla)v) = -\nabla p + \mu \nabla 2v + f$ , The equations allow for the smooth analysis of fluid movement which can be used in numerous other domains allowing for seamless progress across physics and engineering (Batchelor,

2000). Provided by (Easttom, 2022) FEM also provides us with tools to analyze structures that need to be optimized for a particular design, these includes buildings, bridges and spacecraft components. In regards to neuroscience, computational models can imitate neuronal activities with relation to brain functions and disorders using the Hodgkin-Huxley equations in the realm of contrived biology (Hodgkin & Huxley, 1952). In the same fashion, differential equations can be used to maximize the effectiveness of a drug by manipulating pharmacokinetics as well as dosage regimens, making

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them efficacious for various medical conditions (Haasler et al., 2021).

The same principles can be applied to mathematics in finance and economics, where computational modeling aids in risk management and market **Black-Scholes** forecasting. The equation, ∂V∂t+12σ2S2∂2V∂S2+rS∂V∂S-rV=0 is essential in relation to an investment option's price and the mathematician's investment strategies (Black & Scholes, 1973). Other computational models also support time-series forecasting, particularly the analysis done using the autoregressive integrated moving average (ARIMA) model which has been combined with deep learning to give better results in the prediction of stock prices (Box & Jenkins, 1970). Probabilistic Monte Carlo simulation methods, which are extensively used in finance, provide portfolio optimization risk assessments, aiding decision-making in a poorer economy (Ballard et al., 2021). In epidemiology, the model Susceptible and Infectious which aims to assist the public health practitioner in predicting the degree to which disease may spread is outline by dtdS =- $\beta$ SI, dIdt= $\beta$ SI- $\gamma$ I (Easttom, 2022). Climate change and resource management are two global issues that these models can help solve (Xiong et al., 2023).

(NWP) Numerical Weather Prediction uses mathematical equations representing the model's governing dynamics to solve its atmosphere problems in order to enhance the forecast of extreme weather and long-range climate predictions (Haasler et al., 2021). Hydrodynamic models study the management of water resources by predicting flood events and improving irrigation practices to achieve sustainable development (Hormuth et al., 2021). The assessment of climate change impacts is undertaken with highlevel carbon emission computer models, integrated with the ocean current and temperature distribution simulation models for evaluation of global ecosystems (Stocker et al., 2013). The combination of artificial intelligence and super-computing technological progress improves sophisticated computing models by providing the possibility for operational control of the process and rapid replay of events in scientific researches and other industries. They continue to contribute to closing the gap between pure mathematics and its applications,

fostering progress in engineering, finance, medicine, and ecology.

# Challenges and Mathematical Limitations in Computational Modeling

The integration of mathematics into computing through design and software development modeling is complex, often requiring iterative and keen balancing of stability, convergence, and accuracy trade-offs with the heuristic complexity involved of a problem. Large systems, especially those within operational research, pose a challenge referred to as the curse of dimensionality, in which highdimensional problems increase the computational cost exponentially (Mostofian et al., 2023). Moreover, drawbacks additional include algorithmic inefficiency that inhibits the viability of highperformance computing for largescale models, such as simulations of climate or evaluations of financial risk (Ballard et al., 2021). In addition, the power of statistical driven theorems lack interpretability and solely relying on purely theoretical models is incapable of accounting for the real-world variability (Drouhot et al., 2023).

# Future Directions in Computational Mathematical Modeling

Further developments in computational mathematical modeling are likely to emerge due to the integration of quantum computing which greatly accelerates the solution of complex optimization and differential equation tasks (Shor, 1994). The adoption of solvers driven by Artificial Intelligence greatly accelerates the computation, with deep learning improving the precision of simulations in engineering and financial mathematics (Haasler et al., 2021). Data assimilation can improve predictive accuracy within real-time adaptive models in applications such as weather forecasting and epidemiological modeling (Hormuth et al., 2021). In addition, topological data analysis is becoming a more powerful and widely used method in the mathematical sciences due to its ability to capture hidden features structures of high dimensional data sets in which such features can be found. This is invaluable in neuroscience, material science, and modeling of biological systems (Ballard et al., 2021). These innovations are expected to facilitate the

interconnection of computational mathematics with real life issues, while increasing the efficiency, scalability, and accuracy of mathematical model in different scientific fields.

### Conclusion

Applied Computational modeling merges the world of mathematics with sophisticated engineering systems, providing new ways of comprehensively addressing challenges in different fields of science and engineering. With the help of AI, numerical analysis, and supercomputing, there is accurate modeling, optimization, and forecasting in fields like physics, engineering, finance, and even epidemiology. Although there are issues like the complexity of the calculations available or some compromise in the stability, the newly developed quantum computing, AI optimizers, and real-time adaptive models completely change the scope of mathematical modeling. Mathematical modeling becomes easier with expectations and frameworks arising from topology, algorithmic geometry, and new methods for understanding multidimensional data: It shifts from defining problems to solving them. The advancements in AI suggest that the interaction of mathematics, computing, and AI will result in even more innovations and will ensure that math models. lead in scientific and practical productivity.

### REFERENCES

- Adcock, B., & Dexter, N. (2021). The gap between theory and practice in function approximation with deep neural networks. SIAM Journal on Mathematics of Data Science, 3(2), 624-655.
- Ballard, T., Palada, H., Griffin, M., & Neal, A. (2021). An integrated approach to testing dynamic, multilevel theory: Using computational models to connect theory, model, and data. Organizational Research Methods, 24(2), 251-284.
- Biswas, M. C., Chakraborty, S., Bhattacharjee, A., & Mohammed, Z. (2021). 4D printing of shape memory materials for textiles: mechanism, mathematical modeling, and challenges. Advanced Functional Materials, 31(19), 2100257.

- Chunarkar-Patil, P., Kaleem, M., Mishra, R., Ray, S., Ahmad, A., Verma, D., ... & Kumar, S. (2024). Anticancer drug discovery based on natural products: From computational approaches to clinical studies. Biomedicines, 12(1), 201.
- Drouhot, L. G., Deutschmann, E., Zuccotti, C. V., & Zagheni, E. (2023). Computational approaches to migration and integration research: promises and challenges. Journal of Ethnic and Migration Studies, 49(2), 389-407.
- Easttom, W. (2022). Modern cryptography: applied mathematics for encryption and information security. Springer Nature.
- El-Emam, M. A., Zhou, L., Shi, W., Han, C., Bai, L., & Agarwal, R. (2021). Theories and applications of CFD-DEM coupling approach for granular flow: A review. Archives of Computational Methods in Engineering, 1-42.

Ghaffari Laleh, N., Loeffler, C. M. L., Grajek, J., Staňková, K., Pearson, A. T., Muti, H. S., ... Kather, J. N. (2022). Classical & mathematical models for prediction of response to chemotherapy and immunotherapy. PLoS computational biology, 18(2), e1009822.

- Haasler, I., Ringh, A., Chen, Y., & Karlsson, J. (2021). Multimarginal optimal transport with a tree-structured cost and the schrodinger bridge problem. SIAM Journal on Control and Optimization, 59(4), 2428-2453.
- Hansen, P. C., Jørgensen, J., & Lionheart, W. R. (Eds.). (2021). Computed tomography: algorithms, insight, and just enough theory. Society for Industrial and Applied Mathematics.
- Hormuth, D. A., Phillips, C. M., Wu, C., Lima, E.
  A., Lorenzo, G., Jha, P. K., ... & Yankeelov, T.
  E. (2021). Biologically-based mathematical modeling of tumor vasculature and angiogenesis via time-resolved imaging data. Cancers, 13(12), 3008.
- Khaleghi, A., Mohammadi, M. R., Shahi, K., & Nasrabadi, A. M. (2022). Computational neuroscience approach to psychiatry: a review on theory-driven approaches. Clinical

Psychopharmacology and Neuroscience, 20(1), 26.

- Liu, H., Zhao, Y., Pishbin, M., Habibi, M., Bashir, M. O., & Issakhov, A. (2022). A comprehensive mathematical simulation of the composite size-dependent rotary 3D microsystem via two-dimensional generalized differential quadrature method. Engineering with Computers, 38(Suppl 5), 4181-4196.
- Liu, W. K., Li, S., & Park, H. S. (2022). Eighty years of the finite element method: Birth, evolution, and future. Archives of Computational Methods in Engineering, 29(6), 4431-4453.
- Llorente, F., Martino, L., Delgado, D., & Lopez-Santiago, J. (2023). Marginal likelihood computation for model selection and hypothesis testing: an extensive review. SIAM review, 65(1), 3-58.
- Mostofian, B., Martin, H. J., Razavi, A., Patel, S., Allen, B., Sherman, W., & Izaguirre, J. A. (2023). Targeted protein degradation: advances, challenges, and prospects for computational methods. Journal of Chemical Information and Modeling, 63(17), 5408-5432.
- Okorie, O., Charnley, F., Russell, J., Tiwari, A., & Moreno, M. (2021). Circular business models in high value manufacturing: Five industry cases to bridge theory and practice. Business Strategy and the Environment, 30(4), 1780-1802.
- Peng, G. C., Alber, M., Buganza Tepole, A., Cannon, W. R., De, S., Dura-Bernal, S., ... & Kuhl, E. (2021). Multiscale modeling meets machine learning: What can we learn?. Archives of Computational Methods in Engineering, 28, 1017-1037.
- Ramstead, M. J., Seth, A. K., Hesp, C., Sandved-Smith, L., Mago, J., Lifshitz, M., ... & Constant, A. (2022). From generative models to generative passages: a computational approach to (neuro) phenomenology. Review of Philosophy and Psychology, 13(4), 829-857.
- Sharafati, A., Haghbin, M., Motta, D., & Yaseen, Z.M. (2021). The application of soft computing models and empirical formulations for hydraulic structure scouring depth simulation: a comprehensive review, assessment and

possible future research direction. Archives of computational methods in engineering, 28, 423-447.

- Sharma, A., Mukhopadhyay, T., Rangappa, S. M., Siengchin, S., & Kushvaha, V. (2022). Advances in computational intelligence of polymer composite materials: machine learning assisted modeling, analysis and design. Archives of Computational Methods in Engineering, 29(5), 3341-3385.
- Smith, R. C. (2024). Uncertainty quantification: theory, implementation, and applications. Society for Industrial and Applied Mathematics.
- Sun, X., Wang, S., Jin, J., Wang, Z., & Gong, F. (2023). Computational methods of mass transport in concrete under stress and crack conditions: a review. Journal of Intelligent Construction, 1(2), 1-20.
- Waters, S. L., Schumacher, L. J., & El Haj, A. J. (2021). Regenerative medicine meets mathematical modelling: developing symbiotic relationships. npj Regenerative Medicine, 6(1), 24.
- Xiong, W., Dong, H., Ye, C., Wang, Z., Zhong, H., Ji, H., ... & Zhang, T. (2023). Iterative
   attom & Rese. preference learning from human feedback: Bridging theory and practice for rlhf under klconstraint. arXiv preprint arXiv:2312.11456.
- Xiong, W., Dong, H., Ye, C., Wang, Z., Zhong, H., Ji, H., ... & Zhang, T. (2023). Iterative preference learning from human feedback: Bridging theory and practice for rlhf under klconstraint. arXiv preprint arXiv:2312.11456.
- Ye, H., Liang, B., Ng, O. L., & Chai, C. S. (2023). Integration of computational thinking in K-12 mathematics education: A systematic review on CT-based mathematics instruction and student learning. International Journal of STEM Education, 10(1), 3.