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MAPPING PROPERTIES FOR CONIC REGION ASSOCIATED WITH GALUE TYPE STRUVE FUNCTION

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Abstract
This paper explores the mapping properties of a particular class of functions, specifically the Galue-type Struve function, within conic regions of the complex plane. Building upon classical findings in complex analysis and mathematical physics, we examine the geometric characteristics, behavior and transformation properties of the Galue-type Struve function under various conformal mappings. The primary aim of this study is to enhance our understanding of how these functions behave within conic regions and to investigate their potential applications in physical modeling, particularly in domains where complex variables and special functions intersect. Our goal is to get necessary conditions for the convolution operator belonging to the classes UVX (k, α), S(k, α) and S_ς^*.

INTRODUCTION

1. Initiation and preliminaries

Supposed that **A** be the class of function of the form

$$m(\mathrm{t}) = t + \sum_{n=0}^{\infty} a_n t^n \quad \mathrm{t} \in \mathrm{U}$$

is analytic & in the open unit disc **D** = $\{t : |t| < 1\}$ and The set of all functions in **A** that are univalent within **D**. is represented by $\mathbf{S_{tr}}$. Assume that the Groups of starlike and convex functions of order β are shown by $\mathbf{S_{tr}}$ (β) and **C** (β) defined as:

$$\mathrm{S_{tr}}\ (\beta) = \left\{m: m \in A \text{ and } R\left(\frac{tm'(\mathrm{t})}{m(\mathrm{t})}\right) > \beta, \mathrm{t} \in \mathbf{U}, \beta \in [0,1)\right\}$$

and

$$\mathrm{C}\ (\beta) = \left\{m: m \in A \text{ and } R\left(1 + \frac{tm''(\mathrm{t})}{m'(\mathrm{t})}\right) > \beta, \mathrm{t} \in \mathbf{U}, \beta \in [0,1)\right\}$$

It shows that

$\mathbf{S_{tr}}$ (0) = $\mathrm{S_{tr}}$ and **C** (0) = **C**.

The above mentioned two classes i.e $\mathbf{S_{tr}}$ and **C** were proposed by Robertson in 1936. We can get more information about these by checking [1] [2]. Thereafter, two other classes named as uniformly convex functions **UVX** and uniformly starlike functions **US**$_{\mathbf{tr}}$**F** were given in 1991 by Goodman. A function f∈Af \in A is said to be uniformly convex if the image of *f (t)* is convex for any circular arc *t* included in the open unit disc, with the centre likewise included. Ma and Minda in 1992, described a new characterization of the class **UVX**, allowing us to achieve subordination results for the family. These subordination results immediately offer strong growth, distortion, rotation, and covering theorems, as well as sharp constraints on the second and third coefficients. They demonstrated a function k in UVX, which, until rotation, is the only extremal function for these situations [3]. In 1993, Ronning studied starlike functions and established upper bounds for the coefficients and the modulus $|f(z)|$ of the functions within this class.[4].

1.1 Definition:

A function $t \in \mathbf{A}$ is **UVX** in D if and only if

$$\mathbf{R}\left(1+\frac{nt''(n)}{t'(n)}\right) > \left|\frac{nt''(n)}{t'(n)}\right|$$

So, it is clear that a function $t \in A$ is **UVX** in the open unit if $1+\frac{nt''(n)}{t'(n)}$ is in the parabolic region.

1.2 Definition:

A function g ∈ **A** is in S following the condition

$$\mathbf{R}\left(\frac{hg'(h)}{g(h)}\right) > \left|\frac{hg'(h)}{g(h)}-1\right|.$$

Now, we describe a new class relating to starlike function and the sub-classes of K—uniformly convex functions of order β. Bharati [5] explained these classes in 1997 & these can be shown as undeer

1.3 Definition

"A function $P \in \mathbf{A}$ is in **UVX** (K, μ) ⇔

$$\mathbf{R}\left(1+\frac{qP''(q)}{P'(q)}\right) > K\left|\frac{qP''(q)}{P'(q)}\right| + \mu, \quad p \in U,$$

Where $0 \le K < \infty$ and $0 \le \mu < 1$.

The class S (k, β) can be defined with usage of Alexander transform as:

1.4 Definition

A function y∈ **UVX** (K, β) ⇔ $zf' \in$ S (k, β).

The Classes C_ς and S_ς^* were expressed by Ponnusamy and Ronning [6] in 1997 and these can be explained as under:

1.5 Definition

$$\left|\frac{nh''(n)}{h'(n)}\right| < \varsigma, \ (n \in U, \varsigma > 0)$$

then $h \in C_\varsigma$.

1.6 Definition

If h∈A and

$$\left|\frac{nh'(n)}{h(n)}-1\right| < \varsigma, \ (n \in U, \varsigma > 0)$$

then $h \in S_\varsigma^*$

1.7 Definition

A class $P_\varphi^{\tau(\eta)}$ was introduced by Swaminathan [7] in 2004. This class will be useful in order to achieve our main results and it is given as under:

If t∈ **A** and satisfies

$$\left|\frac{(1-\varepsilon)\frac{p(q)}{q}+\varepsilon p'(q)-1}{2\tau(1-\eta)+(1-\varepsilon)\frac{p(q)}{q}+\varepsilon p'(q)-1}\right| < 1$$

Where $\varepsilon \in [0,1)$, $\eta < 1$ and except for zero(0), τ belongs to the complex numbers.

hence $t \in P_\varphi^{\tau(\eta)}$

1.8 Remark

When $\tau = e^{i\theta}\cos\theta$, for $\theta \in (\frac{-\pi}{2}, \frac{\pi}{2})$, then the class $P_\varphi^{\tau(\eta)}$ is also defined as

$$P_\varphi^{\tau(\eta)} = \left\{ g \in A : R\{e^{i\sigma}(1-\varphi)\frac{g(z)}{z}+\varphi g'(z)-\eta \right\} > 0, \sigma \in R$$

1.9 Definition

Struve functions, indicated by "Hv(z)" are special functions that arise as solutions to Bessel's differential equation with non-integer order. They are used in many domains of physics, including acoustics, electromagnetics, and quantum mechanics. The generalized Struve function is the particular solution of

$$z^2w''(z) + bzw'(z) + [cz^2 - v^2 + (1-b)v]w(z) = \frac{4(z/2)^{v+1}}{\sqrt{\pi}\,\Gamma(v+\frac{b}{2})}$$

here b, c, υ ∈ C.

1.10 Definition

The Galue type Struve function is a generalization of the standard Struve function that was proposed to widen its reach under specific boundary conditions and to include extra parameters that affect its asymptotic behavior and singularity structure.

In this study, we focus at the mapping properties of the Galue type Struve function in a conic region of the complex plane. A conic region is often defined as the region bordered by a cone-shaped domain in which geometry is crucial in the mapping and transformation behavior of analytic functions.

Galue type struve function is given as under:

$$ {}_{(\alpha,\lambda,\mu,\xi)}G_{(p,l,m)}(t)=\sum_{i=1}^{\infty}\frac{(-m)^i (t/2)^{2i+p+1}}{\Gamma(\lambda n+\mu)\Gamma\left(\alpha i+\frac{p}{\xi}+\frac{l+2}{2}\right)}. $$

& in class **A**, it can be defined as

$$ G_{(p,q,r)}(m)=m+\sum_{i=2}^{\infty}\frac{\left(\frac{-r}{4}\right)^{i-1}}{(\mu)_{\lambda(i-1)}(\gamma)_{\upsilon(i-1)}}.m^n $$

1.11 Definition

The Hadamard or convolution of the functions of class **A** is defined by

$(f*t)(h) = h + \sum_{n=2}^{\infty} a_n b_n h^n$. ($h \in \boldsymbol{U}$)

Here $f(h)$ and $t(h)$ are convergent power series in **U**.

Now,

$$ G_{p,j,g,\varsigma}(b) = b + \sum_{x=2}^{\infty}\frac{\left(\frac{-j}{4}\right)^{x-1}}{(\sigma)_{\lambda(x-1)}(\varpi)_{\upsilon(x-1)}}.b^x $$

$$ f(b) = b + \sum_{x=2}^{\infty} y_x b^x $$

$$ L(b) = G_{p,j,g,\varsigma}(b) * f(b) = $$

$$ b + \sum_{x=2}^{\infty}\frac{\left(\frac{-j}{4}\right)^{x-1}}{(\sigma)_{\lambda(x-1)}(\varpi)_{\upsilon(x-1)}}.y_x b^x $$

$$ = b + \sum_{x=2}^{\infty} Y_x b^x $$

Where

$$ Y_x = \frac{\left(\frac{-j}{4}\right)^{x-1}}{(\sigma)_{\lambda(x-1)}(\varpi)_{\upsilon(x-1)}}.y_x $$

In order to demonstrate our primary findings, we require the following lemmas.

1.12 Lemma: [8] A function $h \in$ **A** is in UVX (β, α) if it assures

$$ \sum_{e=2}^{\infty} e\{e(1+\beta)-(\beta+\alpha)\}|ae| \le 1-\alpha $$

1.13 Lemma: A function $t \in$ **A** is in S (η, α) if it assures

$$ \sum_{b=2}^{\infty}\{b(1+\varphi)-(\varphi+\mu)\}|ab| \le 1-\mu $$

1.14 Lemma:[9] If $f \in$P,

$$ |an| \le \frac{2|\tau|(1-\xi)}{1+\varphi(n-1)} $$

1.15 Lemma: [10] If g ∈ A and satisfy

$$ \sum_{e=2}^{\infty}(\zeta+e-1)|ae| \le \zeta,\ \zeta>0 $$

1.16 Lemma: If $f \in$ A, satisfies

$$ \sum_{m=2}^{\infty} m(\zeta+m-1)|am| \le \zeta,\ \zeta>0 $$

2. Main Findings:

The main findings are the connections between different subclasses of analytic functions using Galue Type Struve functions.

2.1 Theorem:

Let $\lambda > -1, \sigma > 0 \, and \, \vartheta \in [0,1)$ with inequality such that

$$ \frac{c\cos\theta(1-\psi)(2+b-\vartheta)}{\varepsilon(2\varpi^{\vartheta}-c)[\frac{\sigma}{2}]_{\lambda}} \le (1-\vartheta)). $$

If $f \in P_{\varepsilon}^{\tau(\eta)}, \varepsilon \in [0,1)$ and $\psi < 1$, then $G_{p,c,g}(b) \in$ UVX (K, ϑ) by convolution operator

Proof:

Suppose

$$ G_{p,c,g}(b) = b + \sum_{x=2}^{\infty}\frac{\left(\frac{-c}{4}\right)^{x-1}}{(\sigma)_{\lambda(x-1)}(\varpi)_{\vartheta(x-1)}}.b^x $$

$$g(b) = b + \sum_{x=2}^{\infty} y_x b^x \quad L(b) = G_{p,c,g}(b) * g(b)$$

$$= b + \sum_{x=2}^{\infty} \frac{\left(\frac{-c}{4}\right)^{x-1}}{(\sigma)_{\lambda(x-1)}(\varpi)_{\vartheta(x-1)}} . y_x b^x$$

$$= b + \sum_{x=2}^{\infty} Y_x b^x$$

Where

$$Y_x = \frac{\left(\frac{-c}{4}\right)^{x-1}}{(\sigma)_{\lambda(x-1)}(\varpi)_{\vartheta(x-1)}} . y_x$$

To express the convolution operator $G_{p,c,g}(b) \in$ **UVX** (K, ϑ) from 1.12 Lemma we will get the result.

$$\sum_{x=2}^{\infty} x\{x(1+b) - (b+\vartheta)\} |Yx| \le 1 - \vartheta$$

Now,

$$\sum_{x=2}^{\infty} x\{x(1+b)-(b+\vartheta)\} . \left| \frac{\left(\frac{-c}{4}\right)^{x-1}}{(\sigma)_{\lambda(x-1)}(\varpi)_{\vartheta(t-1)}} . y_x \right|$$

Since $|y_x| \le \dfrac{2\cos\theta(1-\psi)}{1+\varepsilon(x-1)}$ $\le$

$$\sum_{x=2}^{\infty} x\{x(1+b)-(b+\vartheta)\} . \frac{(c)^{x-1}}{4^{x-1}(\sigma)_{\lambda(x-1)}(\varpi)_{\alpha(x-1)}} . \frac{2\cos\theta(1-\psi)}{1+\varepsilon(x-1)}$$

Since

$$\frac{x}{1+\varepsilon(x-1)} \le \frac{1}{\varepsilon} \; \forall x \ge 2, (\sigma)_{\lambda(x-1)} \ge \left(2\left[\frac{\sigma}{2}\right]\right)_{\lambda}(x)$$

and

$$(\varpi)_{\vartheta(x-1)} \ge \varpi^{\vartheta(x-1)},$$

Also $\dfrac{1}{x} \le 2^{x-1} \le$

$$\frac{2\cos\theta(1-\psi)}{\varepsilon}$$

$$\sum_{x=2}^{\infty} \{x(1+b)-(b+\vartheta)\} . \frac{(c)^{x-1}}{4^{x-1}\left(2\left[\frac{\sigma}{2}\right]\right)_{\lambda}(x)(\varpi)^{\vartheta(x-1)}} .$$

$$\le \frac{\cos\theta(1-\psi)}{\varepsilon} . \frac{1}{\left[\frac{\sigma}{2}\right]_{\lambda}}$$

$$\left[(1+b)\sum_{x=2}^{\infty} \frac{x(c)^{x-1}}{4^{x-1}(x)(\omega)^{\vartheta(x-1)}} - (b+\vartheta).\sum_{x=2}^{\infty} \frac{(c)^{x-1}}{4^{x-1}(x)(\omega)^{\vartheta(x-1)}}.\right]$$

$$\le \frac{\cos\theta(1-\psi)}{\varepsilon} . \frac{1}{\left[\frac{\sigma}{2}\right]_{\lambda}}$$

$$\left[(1+b)\sum_{x=2}^{\infty} \left(\frac{c}{4\omega^{\vartheta}}\right)^{x-1} - (b+\vartheta).\sum_{x=2}^{\infty} \frac{(c)^{x-1} 2^{x-1}}{4^{x-1}(\omega)^{\vartheta(x-1)}}.\right]$$

$$\le \frac{\cos\theta(1-\psi)}{\varepsilon} . \frac{1}{\left[\frac{\sigma}{2}\right]_{\lambda}}$$

$$\left[(1+b)\sum_{x=2}^{\infty} \left(\frac{c}{4\omega^{\vartheta}}\right)^{x-1} - (b+\vartheta).\sum_{x=2}^{\infty} \left(\frac{c}{2\varpi^{\vartheta}}\right)^{x-1}.\right]$$

$$\le \frac{\cos\theta(1-\psi)}{\varepsilon} . \frac{1}{\left[\frac{\sigma}{2}\right]_{\lambda}}$$

$$\sum_{x=2}^{\infty} \left(\frac{c}{2\omega^{\vartheta}}\right)^{x-1} \left[(1+b)\left(\frac{1}{2}\right)^{x-1} - (b+\vartheta)\right]$$

$$\le \frac{\cos\theta(1-\psi)}{\varepsilon} . \frac{1}{\left[\frac{\sigma}{2}\right]_{\lambda}}$$

$$\left(\frac{c}{2\varpi^{\vartheta} - c}\right)[2(1+b) - (b+\vartheta)]$$

$$\le \frac{c\cos\theta(1-\psi)}{\varepsilon(2\omega^{\vartheta} - c)} \frac{(2+b-\vartheta)}{\left[\frac{\sigma}{2}\right]_{\lambda}}$$

$\leq 1-\vartheta$

2.2 Theorem:

Assume that $\lambda > -1, \chi > 0 \, and \, \upsilon \in [0,1)$ with inequality such that $\frac{c\cos\partial(1-\gamma)(2+m-\upsilon)}{\varphi(\delta^{\upsilon}-c)[\frac{\chi}{2}]_{\lambda}} \leq (1-\upsilon)$.

If $f \in P_{\varphi}^{\tau(\eta)}, \varphi \in [0,1)$ and $\gamma < 1$, then finally convolution operator $G_{p,g,\varsigma}(h) \in S(\mathrm{K}, \upsilon)$

Proof:

$$G_{p,c,g,\varsigma}(h) = h + \sum_{r=2}^{\infty} \frac{\left(\frac{-c}{4}\right)^{r-1}}{(\chi)_{\lambda(r-1)}(\delta)_{\upsilon(r-1)}}.b^{r}$$

g(h) = $h + \sum_{r=2}^{\infty} y_r h^r$ $L(h) = G_{p,g,\varsigma}(h) * g(h)$

$$= h + \sum_{r=2}^{\infty} \frac{\left(\frac{-c}{4}\right)^{r-1}}{(\chi)_{\lambda(r-1)}(\delta)_{\upsilon(r-1)}}.y_r h^r$$

$$= h + \sum_{r=2}^{\infty} Y_r h^r$$

Where, $Y_i = \frac{\left(\frac{-c}{4}\right)^{r-1}}{(\chi)_{\lambda(r-1)}(\delta)_{\upsilon(r-1)}}.y_r$

We will prove our theorem by using 1.13 Lemma we will prove that

$$\sum_{r=2}^{\infty}\{r(1+\upsilon)-(m+\upsilon)\}\,|Y_r| \leq 1-\upsilon$$

Now,

$$\sum_{r=2}^{\infty}\{r(1+m)-(m+\upsilon)\}.\left|\frac{\left(\frac{-c}{4}\right)^{r-1}}{(\chi)_{\lambda(r-1)}(\delta)_{\upsilon(r-1)}}.y_r\right|$$

Since $|y_r| \leq \frac{2\cos\partial(1-\gamma)}{1+\varphi(r-1)} \leq$

$$\sum_{r=2}^{\infty}\{r(1+m)-(m+\upsilon)\}.\frac{(c)^{r-1}}{4^{r-1}(\chi)_{\lambda(r-1)}(\delta)_{\upsilon(r-1)}}.\frac{2\cos\partial(1-\gamma)}{1+\varphi(r-1)}$$

Since

$$\frac{r}{1+\varphi(r-1)} \leq \frac{1}{\varphi}\ \forall r \geq 2, (\chi)_{\lambda(r-1)} \geq \left(2\left[\frac{\chi}{2}\right]\right)_{\lambda}(r)$$

and

$$(\delta)_{\upsilon(r-1)} \geq \delta^{\upsilon(r-1)}$$

Also $\frac{1}{r} \leq 2^{r-1} \leq \frac{2\cos\partial(1-\gamma)}{r\varphi}$

$$\sum_{r=2}^{\infty}\{r(1+m)-(m+\upsilon)\}.\frac{(c)^{r-1}}{4^{r-1}\left(2\left[\frac{\chi}{2}\right]\right)_{\lambda}(r)(\delta)^{\upsilon(r-1)}}.$$

$$\leq \frac{2\cos\partial(1-\gamma)}{\varphi}.\frac{1}{\left(2\left[\frac{\chi}{2}\right]\right)_{\lambda}}$$

$$\left[\left(\frac{r(1+m)}{r}\right)\sum_{r=2}^{\infty}\frac{(c)^{r-1}}{4^{r-1}(r)(\delta)^{\upsilon(r-1)}}-(m+\upsilon).\sum_{r=2}^{\infty}\frac{(c)^{r-1}}{4^{r-1}(r^2)(\delta)^{\upsilon(r-1)}}.\right]$$

$$\leq \frac{\cos\partial(1-\gamma)}{\varphi}.\frac{1}{\left[\frac{\chi}{2}\right]_{\lambda}}$$

$$\left[(1+m)\sum_{r=2}^{\infty}\left(\frac{2c}{4\delta^{\upsilon}}\right)^{r-1}-(m+\upsilon).\sum_{r=2}^{\infty}\frac{(c)^{r-1}\,2^{2(r-1)}}{4^{r-1}(\delta)^{\upsilon(r-1)}}.\right]$$

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$$\le \frac{\cos\partial(1-\gamma)}{\varphi}\cdot\frac{1}{\left[\frac{\chi}{2}\right]_\lambda}\left[1+m)\sum_{r=2}^{\infty}\left(\frac{c}{2\delta^{\upsilon}}\right)^{r-1}-(m+\upsilon).\sum_{r=2}^{\infty}\left(\frac{c}{\delta^{\upsilon}}\right)^{r-1}.\right]$$

$$\le \frac{\cos\partial(1-\gamma)}{\varphi}\cdot\frac{1}{\left[\frac{\chi}{2}\right]_\lambda}\sum_{r=2}^{\infty}\left(\frac{c}{\delta^{\upsilon}}\right)^{r-1}\left[(1+m)\left(\frac{1}{2}\right)^{r-1}-(m+\upsilon)\right]$$

$$\le \frac{\cos\partial(1-\gamma)}{\varphi}\cdot\frac{1}{\left[\frac{\chi}{2}\right]_\lambda}\left(\frac{c}{\delta^{\upsilon}-c}\right)[2(1+m)-(m+\upsilon)]$$

$$\le \frac{\cos\partial(1-\gamma)}{\varphi}\cdot\frac{1}{\left[\frac{\chi}{2}\right]_\lambda}\left(\frac{c}{\delta^{\upsilon}-c}\right)[2+m-\upsilon]$$

$$\le \frac{c\cos\partial(1-\gamma)}{\varphi(\delta^{\upsilon}-c)}\frac{(2+m-\upsilon)}{\left[\frac{\chi}{2}\right]_\lambda}$$

$$\le 1-\upsilon$$

Hence proved.

Theorem 2.3:
$Let \lambda > -1, \nu > 0 \, and \, \rho \in [0,1)$ with inequality such that $\frac{c\cos\,\text{?}(1-\delta)(1+\Phi)}{\varphi(\varepsilon^{\rho}-c)[\frac{\nu}{2}]_\lambda} \le \Phi$). If $f \in P_{\varphi}^{\tau(\eta)}, \varphi \in [0,1)$ $\delta < 1$ and $\xi < 0$ then the convolution operator $G_{p,g,\varsigma}(N) \in S_{\xi}^{*}$

Proof:
Consider

$$G_{p,b,g,\varsigma}(N) = n + \sum_{a=2}^{\infty}\frac{\left(\frac{-c}{4}\right)^{a-1}}{(\nu)_{\lambda(a-1)}(\gamma)_{\rho(a-1)}}.n^{a}$$

$$g(N) = n + \sum_{a=2} y_a n^a$$

$$L(N) = G_{p,b,g,\varsigma}(N) * g(N)$$

$$= n + \sum_{a=2}^{\infty}\frac{\left(\frac{-c}{4}\right)^{a-1}}{(\nu)_{\lambda(a-1)}(\gamma)_{\rho(a-1)}}.y_a n^{a}$$

$$= n + \sum_{a=2}^{\infty} Y_a n^a$$

Where

$$Y_a = \frac{\left(\frac{-c}{4}\right)^{a-1}}{(\nu)_{\lambda(a-1)}(\gamma)_{\rho(a-1)}}.y_a$$

Now, by using 1.15 Lemma

$$\sum_{a=2}^{\infty}\{\Phi+a-1\}|Y_a| \le \Phi$$

Now,

$$\sum_{a=2}^{\infty}\{\Phi+a-1\}.\left|\frac{\left(\frac{-c}{4}\right)^{a-1}}{(\nu)_{\lambda(a-1)}(\gamma)_{\rho(a-1)}}.y_a\right| \le \Phi$$

Since $|y_a| \le \frac{2\cos\,\text{?}(1-\delta)}{1+\varphi(a-1)}$

$$\le \sum_{a=2}^{\infty}\{\Phi+a-1\}.\frac{(c)^{a-1}}{4^{a-1}(\nu)_{\lambda(a-1)}(\gamma)_{\rho(a-1)}}.\frac{2\cos\,\text{?}(1-\delta)}{1+\varphi(a-1)}$$

Since

$$\frac{a}{1+\varphi(a-1)} \le \frac{1}{\varphi}\ \forall a \ge 2, (\nu)_{\lambda(a-1)} \ge \left(2\left[\frac{\nu}{2}\right]\right)_\lambda (a)$$

and

$$(\gamma)_{\rho(a-1)} \geq \gamma^{\rho(a-1)} \leq$$

$$\frac{2\cos \mathbb{C}(1-\delta)}{a\varphi}$$

$$\sum_{a=2}^{\infty} \{\Phi + a - 1\} . \frac{(c)^{a-1}}{4^{a-1}\left(2\left[\frac{\nu}{2}\right]\right)_{\lambda}(a)(\gamma)^{\rho(a}}$$

$$\leq \frac{\cos \mathbb{C}(1-\delta)}{\varphi} . \frac{1}{\left[\frac{\nu}{2}\right]_{\lambda}}$$

$$\left[(\sum_{a=2}^{\infty}\left(\frac{c}{\gamma^{\rho}}\right)^{a-1} . \left(\frac{1}{2}\right)^{a-1} + (\Phi - 1) . \sum_{a=2}^{\infty}\left(\frac{c}{\gamma^{\rho}}\right)^{a-1} . \right]$$

$$\leq \frac{\cos \mathbb{C}(1-\delta)}{\varphi} . \frac{1}{\left[\frac{\nu}{2}\right]_{\lambda}}$$

$$\sum_{a=2}^{\infty}\left(\frac{c}{\gamma^{\rho}}\right)^{a-1}\left[\left(\frac{1}{2}\right)^{a-1} + (\Phi - 1)\right]$$

$$\leq \frac{c\cos \mathbb{C}(1-\delta)}{\varphi(\gamma^{\rho} - c)} \frac{(2 + \Phi - 1)}{\left[\frac{\nu}{2}\right]_{\lambda}}$$

$$\leq \Phi$$

Which is our required result.

Conclusion

When applied to conic regions of the complex plane, the Galue type Struve function shows intriguing and complex mapping properties. We got a complete understanding of how this function works within such geometrically restricted domains by investigating its asymptotic behavior, boundary transformations, and potential conformal mapping characteristics. Additional research could investigate the particular conditions that lead the Galue type Struve function to become conformal, as well as provide more nuanced insights into its applications in mathematical physics.

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